Translation:Attempt of a Theory of Electrical and Optical Phenomena in Moving Bodies/Definitions

1 Some definitions and mathematical relations

§ 4. *a.* We want to say, that a rotation in a plane *corresponds* to a certain direction of the perpendicular, and namely it shall be the direction into that side, at which an observer must be located, so that for him the rotation is counter-clockwise.

b. The mutually perpendicular coordinate axes OX, OY, OZ are chosen by us, so that the direction of OZ corresponds to a rotation around a right angle of OX to OY.

c. A space, a surface and a line we denote by the letters τ , σ and s throughout, and infinitely small parts by $d\ \tau$, $d\ \sigma$ and $d\ s$.

The perpendicular to a surface will by sketched by *n*, and is always drawn into a certain side, the "positive" one. As regards the line, a certain direction will be called "positive", and namely we note, when we are dealing with the border line *s* of a surface σ , the following rule: If *P* is a fixed point of σ , very near to *s*, and if a second point *Q* traverses the nearest part of *s* in positive direction, then the rotation of *PQ* shall correspond to the direction of the perpendicular to σ .

As regards a closed surface, the *outer side* shall be positive.

d. Usually we denote vectors by German letters; these sometimes also serve to denote the magnitude only. By $\mathfrak{A}_l$ we understand the component of the vector $\mathfrak{A}$ into the direction *l*; by $\mathfrak{A}_x, \mathfrak{A}_y, \mathfrak{A}_z$ therefore the components into the axis-directions.

For a vector with components X, Y, Z we sometimes also write (X, Y, Z) .

e. If ϕ is a scalar magnitude, then we understand by $\dot{\phi}$ the derivative with respect to time *t*. The letter $\dot{\mathfrak{A}}$ denotes a vector with components: $\dot{\mathfrak{A}}_x, \dot{\mathfrak{A}}_y, \dot{\mathfrak{A}}_z$, or $\frac{\partial \mathfrak{A}_x}{\partial t}$ etc.

f. The expression

$$\int \mathfrak{A}_n\ d\ \sigma$$

we call the "integral of vector $\mathfrak{A}$ over the surface σ ", and the magnitude

$$\int \mathfrak{A}_s\ d\ s$$

the "line integral of line *s*".

g. If a vector $\mathfrak{A}$ in any point of space is given, then

$$\frac{\partial \mathfrak{A}_x}{\partial x} + \frac{\partial \mathfrak{A}_y}{\partial y} + \frac{\partial \mathfrak{A}_z}{\partial z}$$

has everywhere a certain value, independent of the choice of coordinate system. We call this magnitude "divergence" of vector $\mathfrak{A}$ and denote it by

$$Div\ \mathfrak{A}\ .$$

For any space limited by a surface σ , the relation is given

$$\int Div\ \mathfrak{A}\ d\ \tau = \int \mathfrak{A}_n\ d\ \sigma,$$

when, as already mentioned, the perpendicular *n* will be drawn into the outside.

h. The magnitudes

$$\frac{\partial \mathfrak{A}_z}{\partial y} - \frac{\partial \mathfrak{A}_y}{\partial z},\ \frac{\partial \mathfrak{A}_x}{\partial z} - \frac{\partial \mathfrak{A}_z}{\partial x},\ \frac{\partial \mathfrak{A}_y}{\partial x} - \frac{\partial \mathfrak{A}_x}{\partial y}$$

can be interpreted as the components of vector $\mathfrak{B}$, which (independent from the choses coordinate system) is defined by the distribution of $\mathfrak{A}$. We call this vector the *rotation* of $\mathfrak{A}$ and denote it by

$$Rot\ \mathfrak{A},$$

and its components occasionally by

$$[Rot\ \mathfrak{A}]_l.$$

If *s* is the border line of surface σ , then we have

Furthermore we will easily find

$$Div\ Rot\ \mathfrak{A} = 0,$$

and for the components of vector $Rot\ Rot\ \mathfrak{A}$

$$\frac{\partial}{\partial x} Div\ \mathfrak{A} - \triangle \mathfrak{A}_x,\ \text{etc.}$$

Here, the letter Δ has, like in all our formulas, the meaning

$$\triangle = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

i. If *m* and *n* are scalar magnitudes, then we attribute to the expressions

$$-\mathfrak{A},\ m\,\mathfrak{A},\ m\,\mathfrak{A} \pm n\,\mathfrak{B}$$

the known meanings.

j. By $[\mathfrak{A}.\mathfrak{B}]$ we understand the so-called "vector product", namely a vector whose magnitude is given by the area of the parallelogram drawn over $\mathfrak{A}$ and $\mathfrak{B}$, and whose direction is perpendicular to the plane that is laid trough $\mathfrak{A}$ and $\mathfrak{B}$, and namely in a way, by that the direction of $\mathfrak{A}$ is transformed into the direction of $\mathfrak{B}$.

As regards the components it can be written $[\mathfrak{A}.\mathfrak{B}]_l$; the components into the axis-directions are:

$$\mathfrak{A}_y\mathfrak{B}_z - \mathfrak{A}_z\mathfrak{B}_y,\ \mathfrak{A}_z\mathfrak{B}_x - \mathfrak{A}_x\mathfrak{B}_z,\ \mathfrak{A}_x\mathfrak{B}_y - \mathfrak{A}_y\mathfrak{B}_x,$$

and

$$[\mathfrak{B}.\mathfrak{A}] = -[\mathfrak{A}.\mathfrak{B}].$$

k. The advantage of the previously introduced expressions mainly consists in the fact, that now three equations like

$$\mathfrak{A}_x = \mathfrak{B}_x,\ \mathfrak{A}_y = \mathfrak{B}_y,\ \mathfrak{A}_z = \mathfrak{B}_z$$

can be summarized in one formula

$$\mathfrak{A} = \mathfrak{B}$$

However, in the course of the investigation we will often use the three individual equations. If they have the same form, so that they transform into one another by cyclic permutation of the letters, then we can restrict ourselves to only writing down the first equation, and to sketch the two others by "etc.".

l. We will often have to consider bodies with molecular structure. Then functions arise, whose value *quickly* changes in the individual molecules and in the interspaces, and namely in a highly irregular way, as the molecules themselves are not always structured and oriented regularly. In those cases it is recommended, to calculate with *averages*, which we define as follows:

We describe around center-point *P* a sphere of area *I*, and calculate for it, when ϕ is the magnitude to be considered, the integral $\int \phi\, d\,\tau$. Then we call

for which we want to write $\bar{\phi}$, the "average of ϕ at point *P*".

If we give to the sphere, where ever *P* may lie, always the same magnitude, then $\bar{\phi}$ can obviously only depend on *t* and the coordinates *x, y, z* of point *P*. It is clear that also $\bar{\phi}$ will show "rapid" changes from point to point, as long the sphere encloses only a few molecules, yet by a continuing increase the changes will step back more and more. We think for once and for all time a certain *R* as chosen, which is just as great that — with respect to the degree of exactitude that can be reached by the observations — we can neglect the rapid changes in $\bar{\phi}$. Then only the slow changes from point to point remain, that are accessible to our senses, and in all real cases they proceed so slow, that they hardly appear in spaces which are considerably greater as the sphere *I*. In these cases, $\bar{\phi}$ will be given only by expression (2), when we don't apply it to the mentioned sphere, but to an arbitrary formed larger space.

Of course $\bar{\phi} = \phi$ everywhere, as soon as ϕ doesn't show rapid changes.

Furthermore we easily find

$$\frac{\partial \bar{\phi}}{\partial t} = \overline{\frac{\partial \phi}{\partial t}},\ \frac{\partial \bar{\phi}}{\partial x} = \overline{\frac{\partial \phi}{\partial x}},\text{w. s. u.}$$

m. By the average of a vector $\mathfrak{A}$ we understand a vector — it may be called $\bar{\mathfrak{A}}$ —, whose components are the averages of $\mathfrak{A}_x, \mathfrak{A}_y, \mathfrak{A}_z$. Consequently we have

$$\dot{\bar{\mathfrak{A}}} = \bar{\dot{\mathfrak{A}}},\ Div\,\bar{\mathfrak{A}} = \overline{Div}\,\bar{\mathfrak{A}},\ Rot\,\bar{\mathfrak{A}} = \overline{Rot}\,\bar{\mathfrak{A}}.$$

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2.1 Text

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